
The applications of Turing (in)computability to classical mathematics

Guido Gherardi*¹

¹Inst. 1 Informatik Universität der Bundeswehr München – Germany

Abstract

What role do Turing machines play today in mathematics?

An apparently substantial difference between lambda-calculus and the Turing machine model is reflected in their applications in the foundations of mathematics. Lambda-calculus is particularly appreciated to extract computational content from constructive proofs. In contrast, (oracle) Turing machines are primarily used in the domain of classical mathematics. Several examples of their applications in classical topology and descriptive set theory establish a strong connection between Borel measurability and Weihrauch reducibility. Other significant examples come from real analysis and concern differentiability. In particular we discuss recent results by Bienvenue, Brattka, Nies, Pathak, Simpson and other authors, as well as older results by Denmuth, which constitute computational formulations in terms of randomness of classical results on differentiability and measure theory. Notwithstanding the computational nature of the different notions of randomness, random numbers are not computable, hence non constructive; moreover the corresponding theories are formulated in terms of classical logic. Surprisingly, it turns out that the notions of randomness and differentiability are intimately related, and this fact sheds a new light on a better comprehension of the original classical results.

*Speaker